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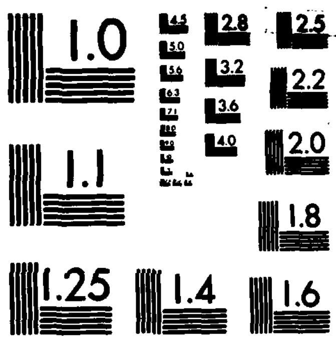

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AFOSR-TR- 86-0371

Markovian Shock Models, Deterioration Processes, Stratified Markov Processes and Replacement Policies

AFOSR 80-0245

by

Department of Mathematics
University of North Carolina at Charlotte
Charlotte, North Carolina

Final Report

Department

of

Mathematics

The University of North Carolina at Charlotte

Charlotte, North Carolina 28223

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS: A174046
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 86-0371
6a. NAME OF PERFORMING ORGANIZATION: University of North Carolina; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION: Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code): Dept. of Mathematics, Charlotte, North Carolina 28223	7b. ADDRESS (City, State and ZIP Code): Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR; 8b. OFFICE SYMBOL (If applicable): NM	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-80-0245
8c. ADDRESS (City, State and ZIP Code): Bldg 410, Bolling AFB DC 20332	10. SOURCE OF FUNDING NOS.: PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2304; TASK NO. A5; WORK UNIT NO.

11. TITLE (Include Security Classification) MARKOVIAN SHOCK MODELS, DETERIORATION PROCESSES, STRATIFIED MARKOV PROCESSES AND REPLACEMENT POLICIES

12. PERSONAL AUTHOR(S) Dr. M. Abdel-Hameed

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Final	FROM 7/1/80 TO 12/31/85		22

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

During the period of this grant, 13 papers were written. Research topics included shock and wear processes, optimal maintenance and replacement policies, positive dependence of components, life distribution properties of devices, analysis of censored failure time data, and accelerated life testing of systems. In 1983, the principal investigators hosted a conference on stochastic failure models.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: BRIAN W. WOODRUFF, Major, USAF	22b. TELEPHONE NUMBER (Include Area Code): (202) 767-5027; 22c. OFFICE SYMBOL: NM

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

The following work has been accomplished under the grants AFOSR-80-0245, 80-0245A, 80-0245B, 80-0245C and 80-0245D.

A). <u>SHOCK AND WEAR PROCESSES.</u>

A device is subject to shocks causing damage. Let, X_t denote the cumulative damage the device suffers during the interval $(0,t]$ plus the initial damage at time 0. It is clear that this will be a right continuous and increasing stochastic process. Every jump of this process has the interpretation of being the damage due to a shock at the time of the jump. Assume that X_t is a nonstationary Levy process. The Poisson random measure generating the Lévy process and describing the shock times and their damage magnitude has a mean measure $\nu(dt,dz)$ of the form $\Lambda(dt)\mu(dz); \int_{R_0} (z \wedge 1)\mu(dz) < \infty$. This process is assumed to be defined on some probability space (Ω,F,P). The device has a certain threshold Y defined on another probability space (Ω,F,P). The device fails when the damage first exceeds the threshold. The failure time ζ defined on $\Omega \times \Omega'$ according to the following:

$\zeta(w,w') = \inf\{t \geq 0 : X_t(w) \geq Y(w')\}$. Let $\overline{G}(y)$ be the probability that threshold Y exceeds or equal to y, $y \geq 0$. Then for $t \geq 0$ and $x \geq 0$ the survival probability $\overline{F}_x(t)$ is given by $\overline{F}_x(t) = E_x[\overline{G}(X_t)]$.

Then, the following holds:

i) if $\overline{G}$ has increasing failure rate and Λ is convex and $\mu << $ leb. with density f that is a Pólya frequency function of order two, then $\overline{F}_x(t)$ has increasing failure rate for each $x \geq 0$;

ii) if $\overline{G}$ has a decreasing failure rate and Λ is concave, then $\overline{F}_x$ has a decreasing failure rate for each $x \geq 0$;

iii) if $\overline{G}$ has increasing failure rate average and Λ is starshaped; then $\overline{F}_x$ has increasing failure rate average for each $x \geqslant 0$;

iv) if $\overline{G}$ has decreasing failure rate average and Λ is antistarshaped $\mu << \text{leb.}$ with PF_2 density, then $\overline{F}_x$ has decreasing failure rate average for each $x \geqslant 0$;

v) if $\overline{G}$ is better than used and Λ is superadditive, then for each $x \varepsilon (0,1)$ and $x,t,s \geqslant 0$; we have

$$\overline{F}_x(t+s) \leqslant \overline{F}_{\alpha x}(t)\overline{F}_{(1-\alpha)x}(s)$$

In particular $\overline{F}_0$ is new better than used.

vi) if $\overline{G}$ is new worse than used and Λ is subadditive, then $\overline{F}_x$ is new worse than used.

Moeover, when $\overline{G}$ depends both on the accumulated damage and time then the following holds:

i) if the map $x \to \overline{G}(x,t)$ has increasing failure rate average for each $t \geqslant 0$, the map $t \to \overline{G}(x,t)$ is decreasing for each $x \geqslant 0$, and Λ is starshaped, then $\overline{F}_x$ has increasing failure rate average for each $x \geqslant 0$;

ii) if the map $x \to \overline{G}(x,t)$ has decreasing failure rate average, the map $t \to \overline{G}(x,t)$ is increasing for each $x \geqslant 0$, Λ is antistarshaped and $\mu << \text{leb.}$ with PF_2 density, then $\overline{F}_0(t)$ has decreasing failure rate average for each $x \geqslant 0$;

iii) if the map $(x,t) \to \overline{G}(x,t)$ satisfied $\overline{G}(x+y,t+s) \leqslant \overline{G}(x,t)\overline{G}(y,s)$ for each $x,y,t,s \geqslant 0$, Λ is superadditive, then for each $x,t,s \geqslant 0$ and $\alpha \varepsilon (0,1)$ we have

$$\overline{F}_x(t+s) \leqslant \overline{F}_{\alpha x}(t)\overline{F}_{(1-\alpha)x}(s).$$

In particular $\overline{F}_0$ is new better than used.

iv) if the map $(x,t) \to \overline{G}(x,t)$ satisfies the property $\overline{G}(x+y,t+s) \geqslant \overline{G}(x,t)\overline{G}(y,s)$ for each $x,y,t,s \geqslant 0$ and Λ is subadditive, then $\overline{F}_x$ is new worse than used for each $x \geqslant 0$.

B). OPTIMAL MAINTENANCE AND REPLACEMENT POLICIES.

Suppose that a device is subject to shocks causing damage. Damage accumulates and the accumulated damage can be described as a Lévy process with man measure $\nu(dt,dz)$ of the form $dt\mu(dz)$. This simply means that the process has stationary independent increments. Assume that the device can be replaced before or at failure. Replacements at failure costs c dollars. If the device is replaced before failure a smaller cost is incurred. The cost depends on the accumulated damage at the time of replacement, and is denoted by $c(.)$. Let X_t denote the accumulated damage till time t. For any Markovian replacement policy τ, the average cost per unit time, ψ_τ is of the form:

$$\psi_\tau(x) = \{E_x[c(x_\tau)I_{(\tau<\zeta)}] + cP_x(\tau \geqslant \zeta)\}/E_x(\tau), \quad \text{for each } x \geqslant 0.$$

Let $c_1(x) = c - c(x)$, $x \geqslant 0; c(+\infty) = 0$ and note that

$$\psi_\tau(x) = \{c - E_x[c_1(X_\tau)]\}/E_x(\tau).$$

A Markovian replacement time τ_x^* is called <u>optimal</u> if $\psi_{\tau_x^*}(x) = \inf \psi_{\tau}(x)$ where the infimum in the right hand side is taken over all Markovian replacement times.

Let $G\,c_1(x)$ denote the infinitisemal generator of the function $c_1(x), b(x) = \inf \psi_\tau(x)$ and assume that $b > 0$. Then under the assumptions of the finiteness of the average life time of the device and suitably chosen conditions on the cost functions, we find that the optimal Markovian replacement policy is a control policy of the form

$$\tau_x^* = \inf\{t \geqslant 0 : X_t \varepsilon [x_x, \infty) \Lambda \zeta,$$

where

$$\alpha_x = \inf\{y : b(x) + Gc_1(x+y) \leqslant 0\}.$$

The results reported on in B and C above have appeared in the paper "Life distributions of devices subject to a Lévy wear process", in <u>Mathematics of Operations Research</u>.

C). <u>POSITIVE DEPENDENCE OF COMPONENTS</u>.

The area of monotone dependence of multivariate distributions has attracted the attention of many authors over the past decade. In this research we investigate covariance inequalities for a class of multivariate strongly unimodal densities, i.e., the class of logarithmically concave densities. Strong unimodal densities play an important role in probability as well as in Statistics since they enjoy the monotone likelihood ratio property.

For $n = 1, 2, \ldots,$ let

$$H_n = \{f : R^n \to R_+ : f \text{ is strongly unimodal and symmetric}\}$$

$$A_n = \{A : A \text{ is } n \times n \text{ diagonal matrix with diagonal elements } \pm 1\}$$

$$G_n = \{f \varepsilon H_n : f(\underline{x}A) = f(\underline{x}) \text{ for all } \underline{x} \varepsilon R^n \text{ and all } A \varepsilon A_n\},$$

$$L_n = \{K : K \text{ is a convex symmetric subset of } R^n\}.$$

We show that for a certain class of unimodal vectors the random variables

$f(\underline{X})$ and $g(\underline{X})$ are positively correlated for all f and g in H_n. It follows in particular that if $\{\underline{X}(t), t \geq 0\}$ is a Wiener process, then for each $t \geq 0$, and each f,g in H_n the random variables $f(\underline{X}(t))$ and $g(\underline{X}(t))$ are positively correlated. We also obtain bounds on the right tail probabilities of random vectors whose densities belong to classes containing the above ones.

Details may be found in "Generating Positively Correlated Random Variables from a Sequence of Independent Random Variables with Symmetric Logarithmically Concave Densities", UNCC-Technical Report (1984)

D). LIFE DISTRIBUTION PROPERTIES OF DEVICES SUBJECT TO A PURE JUMP DAMAGE PROCESS.

Suppose that a device is subject to damage, the amount of damage it suffers, over time, is assumed to be an increasing pure jump process. We denote such a process by $X \equiv (X_t, t \geq 0)$.

Çinlar and Jacob (1981) showed that there exists a Poisson random measure on $R_+ \times R_+$ whose mean measure at the point (s,z) is $dsdz/z^2$ and a deterministic function c defined on the positive quadrant that is increasing in the second argument such that

$$\sum_{s \leq t} f(X_{s-}, X_s) = \int_{[0,t] \times R_+} N(ds,dz) f(X_{s-} + c(X_{s-}, f))$$

almost everywhere for each function f on $R_+ \times R_+$ with $f(x,x) = 0$ for all x in R_+. In particular, it follows that

$$X_t = X_0 + \int_{[0,t] \times R_+} N(ds,dz) c(X_{s-}, z).$$

The above formula has the following interpretation

$t \to X_t(w)$ jumps at s if the Poisson random measure $N(w,.)$ has an atom (s,z) and then the jump is from the left-hand limit $X_{s-}(w)$ to the right-hand limit

$$X_s = X_{s-} + c(X_{s-}, z).$$

The function $c(x,z)$ represents the damage due to a shock of magnitude z occuring at a time when the previous cumulative damage is equal to x. Assume that the device has a threshold Y and it fails once the damage exceeds or equal to Y. The failure time is therefore given by

$$\zeta = \inf\{t : X_t \geqslant Y\}.$$

Let $\overline{G}$ be the right tail probability of the random variable Y. Then the survival function is given by, for $t \geqslant 0$,

$$\begin{aligned} S(t) &\equiv P(S \geqslant t) \\ &= E[\overline{G}(X_t)] \end{aligned}$$

We show that life distribution properties of $\overline{G}$ are inherited as corresponding properties of S. The following are samples of some of the results obtained:

(1) Theorem. Suppose that the function c above satisfies the following condition

$$c(.,z) : R_+ \to R_+ \quad \text{is increasing for each} \quad z \geqslant 0.$$

Then

i) S has increasing failure rate when $\overline{G}$ has increasing failure rate and X has a totally positive density of order two;

ii) S has increasing failure rate on the average when $\overline{G}$ has an increasing failure rate on the average and X has a totally positive density of order two;

iii) S is new better than used if $\overline{G}$ is new better than used.

(2) Theorem. Suppose that the function c satisfies the following condition

$$c(.,z):R_+ \to R_+ \quad \text{is decreasing for each} \quad z \geq 0.$$

Then

i) S has decreasing failure rate when $\overline{G}$ has decreasing failure rate and X has a totally positive density of order two.

ii) S has a decreasing failure rate average when $\overline{G}$ has a decreasing failure rate average and X has a totally positive density of order two.

iii) S is new worse than used when $\overline{G}$ is new worse than used and the function $x \to x + c(x,z)$ is an increasing function for each $z \geq 0$.

(3) Theorem.

i) Suppose that $\overline{G}(.,t)$ has increasing failure rate average for each $t \geq 0$, the mapping $\overline{G}(x,.)$ is a decreasing function for each $x \geq 0$, X has a totally positive density of order two, and $c(.,z)$ is an increasing function for each $z \geq 0$. Then S has increasing failure rate average.

ii) Suppose that $\overline{G}(.,t)$ has a decreasing failure rate average for each $t \geq 0$, the mapping $\overline{G}(x,.)$ is an increasing function for each $x \geq 0$, X has a totally positive density of order two and $c(.,z)$ is a decreasing function for each $z \geq 0$. Then S has a decreasing failure rate average.

iii) Suppose that $V = -\ln \overline{G}$ is a superadditive function on R_+^2 and $c(.,z)$ is an increasing function for each $z \geq 0$. Then S is new better than used.

iv) Suppose that V above is a subadditive function on R_+^2 while the function $x \to x + c(x,z)$ is decreasing for each $z \geq 0$. Then S is new worse than used.

We also discuss the optimal replacement problem for such devices. Define, for $t \geq 0$,

$$Z_t = \begin{cases} X_t , & t \leq \zeta \\ +\infty , & t \geq \zeta . \end{cases}$$

The process $Z = (Z_t)$ is obtained by killing the process X at the failure time of the device.

A device subject to the damage process Z can be replaced before or at failure. Each replacement at failure costs c dollars, $c > 0$. The cost of a replacement before failure depends on the damage level at the time of replacement and is denoted by $c(.)$. That is to say, $c(x)$ is the cost of a replacement when the damage level at time of replacement is equal to x. Naturally, we assume that $c(x)$ is increasing and bounded above by c. Let (H_t) be the canonical history of Z. Moreover, U denotes the class of stopping times that do not exceed the life time ζ.

For any stopping time τ in U we let ξ denote the expected cost of replacement per unit time. We are interested in finding the stopping time τ^* in U satisfying

$$\xi_{\tau^*} = \inf_{\tau \in U} \xi_\tau$$

That is to say, we want to find the stopping time in U that minimizes the expected cost per unit time over U. We call such stopping time the optimal replacement time. We give conditions on the cost function $c(x)$ and the damage function $c(x,z)$ that guarantee that the optimal replacement policy is a control-limit policy.

The above results have appeared in the paper entitled "Life distribution properties of devices subject to pure jump damage process", Journal of Applied Probability.

E). A POWER TRANSFORMATION EXPONENTIAL REGRESSION MODEL FOR CENSORED FAILURE TIME DATA.

Suppose that we have n items which are subject to failure. Let $T_1^*, T_2^*, \ldots, T_n^*$ be the random variables representing the failure time of the first, second, ..., n th item respectively. We assume that right censoring may occur because of the need for early termination of the experiment and let $T_1, T_2, \ldots, T_n$ represent the recorded survival times. Defining censoring indicator variables

$$w_i = \begin{cases} 1 & \text{if } T_i^* \text{ is uncensored} \\ 0 & \text{if } T_i^* \text{ is censored} \end{cases}$$

we have

$$T_i^* = T_i \quad \text{if} \quad w_i = 1 \quad \text{and} \quad T_i^* > T_i \quad \text{if} \quad w_i = 0.$$

We let

$$n_u = \sum_{i=1}^{n} w_i \quad \text{and} \quad n_c \sum_{i=1}^{n} (1 - w_i)$$

denote the numbers of uncensored and censored observations respectively. Without loss of generality we label the individuals such that the first n_u items have uncensored times to failure and the rémaining n_c have censored times to failure.

We now suppose that measurements are available on k explanatory variables $X_1, X_2, \ldots, X_k$. Setting $\underset{\sim}{x}' = (x_1, x_2, \ldots, x_k)$, the probability density function and survival function of T^* given $\underset{\sim}{x}$ are denoted by $f(t;\underset{\sim}{x})$ and $S(t;\underset{\sim}{x})$ respectively. If the failure rate does not depend on t, for any given $\underset{\sim}{x}$, T^* has the exponential distribution with probability density function

$$F(t;x) = \begin{cases} \mu_{\underset{\sim}{x}}^{-1} \exp(-t/\mu_{\underset{\sim}{x}}), & t > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Various models have been proposed in the literature to represent the dependence of $\mu_{\underset{\sim}{x}}$ on $\underset{\sim}{x}$. Fiegel and Zelew (1965)
Consider the model from

$$\mu_{\underline{x}} = (1 + \underline{x}'\beta)$$

while Greenberg et al (1974) use the form

$$\mu_{\underline{x}} = \lambda/(1 + \underline{x}'\beta)$$

where $\underline{\beta}' = (\beta_1, \ldots, \beta_k)$ and λ is a positive constant. Both models require that the condition $\underline{x}'\underline{\beta} > -1$ must be imposed to insure that $\mu_{\underline{x}} > 0$. An alternative model which does not require a constraint to be imposed on $\underline{x}'\underline{\beta}$ is

$$\mu_{\underline{x}} = \lambda \exp(\underline{x}'\underline{\beta})$$

This model arises for the exponential case from the well-known family of proportional hazard regression models (see e.g. Kay (1977)) in which an assumed underlying hazard function is adjusted by multiplicative exponentail factors to allow for the effect of the explanatory variables, Prentice (1973), also discusses the use of censored regression models for the exponential case.

In this paper, we consider the power transformation model given by

$$\mu_{\underline{x}} = \lambda(1 + \delta\, \underline{x}'\underline{\beta})^{1/\delta}.$$

We refer to δ as the power parameter. It is seen that when $\delta = 1$, the model corresponds to the model used by Fiegel and Zelew, while if $\delta = -1$ the model proposed by Greenberg et al is obtained after appropriate reparameterisation. When $\delta \to 0$ the exponential model for $\mu_{\underline{x}}$ given above is obtained.

In general, the power parameter δ as well as the coefficient vector β will have to be estimated from the data. We obtain maximum likelihood estimators for these parameters and it is shown how the estimates can be obtained using the statistical package G LIM. We also discuss the assessment of the goodness of fit of the model and numberical examples are given to illustrate the procedure.

Details may be found in "A Power Transformation Regression Model for Censored Failure Time Data", UNCC - Technical report (1984).

F). CONSERVATIVE AND DISSIPATIVE PARTS OF NON-MEASURE PRESERVING WEIGHTED COMPOSITION OPERATORS.

Earlier joint work with A. Lambert and T. Hoover on measure preserving weighted composition operators is extended. The main result of this paper implies that, if $\tau: X \to X$ is a measurable transformation of a probability space X onto X which is measure isomorphic to a measure preserving transformation $\pi: X \to X$ then there exists a weight function $\phi: X \to (0, \infty)$ such that the operator $[T_{\phi,\tau} f](x) = \phi(x) f \circ \tau(x)$ is conservative. Using this result, it is possible to extend many results which were previously known only for measure preserving weighted composition operators to the class of weighted composition operators whose composition part is isomorphic to a measure preserving transformation. In particular, the recent characterization of the point spectrum for measure preserving weighted composition operators due to A. Lambert, has a complete analogue for this wider class. These results have appeared in the paper "Conservative and dissipative parts of nonmeasure preserving weighted composition operators", Houston J. Math, 8, 575 - 586, (1982).

G). STABILITY OF OPTIMAL STOPPING TIMES FOR MARKOV CHAINS AND THEIR APPLICATIONS TO OPTIMAL REPLACEMENTS.

With respect to this discussion, a Markov Chain is a discrete time, homogeneous, nonterminating Markov Process with values in a state space $(E,\mathcal{E})$. Two Markov chains $X^1 = (\Omega, x_n, F_n, P^1_x)$ and $X^2 = (\Omega, x_n, F_n, P^2_x)$ are said to be (ε, F_m)-close if, for each $x \varepsilon E$ and $G \varepsilon F_m$,

$(1-\varepsilon)P^1_x(G) \leqslant P^2_x(G) \leqslant (1+\varepsilon)P^1_x(G)$ and $(1-\varepsilon)P^2_x(G) \leqslant P^1_x(G) \leqslant (1+\varepsilon)P^2_x(G)$

where $m \varepsilon \{0,1,2,\ldots,\infty\}$ and ε is in $[0,1)$. If G is a class of bounded, real valued reward functions and M is a class of Markov times, then X is said to be (F_m, G, M) - stable $(c(F_m, G, M)$ - stable) if for each $\alpha > 0$ and $g \varepsilon G$, there exists an $\varepsilon > 0$ and a $\beta_0 > 0$ such that, if $X^1 = (\Omega, x_n, F_n, P^1_x)$ is (ε, F_m)-close to X, then for any $0 > \beta \leqslant \beta_0$, $\tau^1_\beta = \inf\{n/g(x_n) \geqslant s^1(x_n) - \beta\}$ $(\tau_\beta = \inf\{n/g(x_n) \geqslant s(x_n) - \beta\}$ is an (α, s)-optimal stopping time for $X((\alpha, s^1)$-optimal stopping time for X^1 here, s and s^1 are payoffs). Weaker forms of stability result if the conclusion holds only for β in some interval bounded away from 0. In the paper, M is either MF, the class of first entry Markov times, or it is, for some $n > 0$, $M(n)$, the class of stopping times bounded by n.

If $m = \infty$ or $M = M(n)$, it is shown that all Markov chains are stable in a very strong sense. If $m < \infty$, then the (F_m, G, MF) types of stability are equivalent to corresponding (F_1, G, MF) types.

Examples are given of chains which are (F_1, G, MF)-stable and $c(F_1, G, MF)$-stable. An example is given of a chain that is weakly (F_1, G, MF)-stable and weakly $c(F_1,G,MF)$-stable but is not $c(F_1,G,MF)$-stable. Finally, an example is given of a chain and a reward function g such that the chain is not stable in any $(F_1,\{g\},MF)$ sense. The most useful theorem states that, if the g is a reward function and the MF-payoff associated with g is constant on the orbits of X, then X is weakly $(F_1,\{g\}MF)$-stable and weakly $c(F_1,\{g\},MF)$-stable.

Details are available in "On the stability of optimal stopping times for Markov Chains, UNCC-Technical Report (1981).

H). STABILITY OF OPTIMAL STOPPING TIME AND OPTIMAL REPLACEMENT PROBLEMS.

The investigations are restricted to stability for Markov processes which are either standard processes in the sense of Bluementhal and Gretoor or are Markov chains. For optimal stopping times, two broad classes of stability are considered. A problem (X,g) is considered to be "c-stable", here X is a process and g is a reward function defined on the state space of X, if a close to optimal solution to the problem (X,g) is close to optimal for any problem (X^1,g_1) which is sufficiently "close" to the problem (X,g). The other broad class of stability considered views (X,g) as "stable" if a clost to optimal solution to the problem (X^1,g_1) is close to optimal for (X,g) provided (X^1,g_1) is "close enough" to (X,g).

One group of results characterizes stability of optimal stopping times in terms of a corresponding but simpler property for the excessive majorants associated with the problems by the theory. For example the following paraphrases the characterization for c-stability. Here and below, if X^i is a process and g_i a reward function, then $\pi_i g_i$ will denote the the smallest excessive majorant of g_i with respect to X^i.

We prove the following:

Let X be a process and let g be E_Δ-measurable, bounded, lower C_0-continuous and, if X does not have finite lifetime, nonnegative and zero at Δ. Then (X,g) is c-stable if and only if, for each $x \in E$ and $\varepsilon > 0$, $\pi_1 \pi g(x) \leq \pi g(x) + \varepsilon$ whenever X^1 is close enough to X. A similar result holds for stability except that one only gets sufficiency and several technical assumptions have to be added. Still, these results represent a considerable reduction of the problem since quite a lot is known about the structure of excessive functions. Also, they can be used directly to show that the one and two dimensional Wiener processes are stable as are the one and two dimensional symmetric random walks. They can also be used to show stability of the classical nonsymmetric random walks on the integers (although, here, somewhat stronger forms of stability are obtainable).

Another class of results concerns stability in the setting of finite lifetimes. The metric used reflects the closeness of the lifetimes and very strong results are obtained. Essentially in this setting one always has both kinds of stability for optimal stopping times. These results are shown to imply the stability of the optimal replacement problem in the event that the expected values of the lifetimes are also finite. Since such an assumption is reasonable, this constitutes a very satisfactory result.

This work is contained in "Stability of optimal stopping times for Markov processes", UNCC - Technical Report (1982).

I). AN ITERATIVE SCHEME FOR APPROXIMATING OPTIMAL REPLACEMENT POLICIES.

Let $X = X_t$, $t \geq 0$ be an arbitrary stochastic process with augmented state space E^Δ and lifetime $\lambda = \inf\{t \mid X_t = \Delta\}$. We analyze the following iterative technique. Let $b_1 = E^0 g(X\lambda)/E^0(\lambda)$. Consider the problem of maximizing the criterion $\psi(b_1,\tau) = b_1 E^0(\tau) - E^0 g(x_\tau)$ for $\tau \leqslant \lambda$. If we are interested in a generalized ε-optimal policy, then tolerances $x > 0$ and $\beta > 0$ are determined in terms of X, g, λ and ε so that, if a β-optimal policy τ_1 for $\psi(b_1,\tau)$ gives $\psi(b_1,\tau) \leqslant \alpha$ then λ is already ε-optimal, otherwise take $b_2 = E^0 g(X_{\tau_1})/E^0(\tau_1)$ and repeat the steps on the criterion $\psi(b_2,\tau) = b_2 E^0(\tau) - E^0 g(X_\tau)$. Under very reasonable assumptions on g (it must be positive and bounded away from zero), it is shown that this iterative method will supply a generalized ε-optimal replacement policy. We further implement this scheme on a computer for certain Markovian damage models. In doing so, the discrete approximations which are required are fully justified and some feeling for the speed of convergence of the procedure is obtained. The iterative scheme itself is very fast. Solving the related optimal stopping problems using dynamic programming techniques is, however, very slow.

These results appeared in "An Iterative Scheme for Approximating Optimal Replacement Policies", Reliability Theory and Models, M.S. Abdel-Hameed, E. Çinlar and J. Quinn, Editors, Academic Press, New York, (1984).

J). <u>ACCELERATED LIFE TESTING OF SYSTEMS WHOSE WEAR IS GOVERNED BY A CONTROLLED ODE.</u>

A device is composed of a material with defects, perhaps received in manufacture. Let x denote the size of a given defect. Ther is a threshold value B for x such that the device fails when $x = B$. The device is used in an environment with m-states $\bar{v} = (s_1, s_2, \ldots, s_m)$. For fixed state vector $\bar{v}$, we assume that defect size grows with time according to an ODE of the form

$$dx/dt = h(x,\bar{v}) > 0 \tag{1}$$

We also assume the existence of a sigma-finite measure n on $(0,B]$ such that

$n(d,B)$ is strictly increasing in d,
$n(d,B) < \infty$ for all $d > 0$, and
if $A \subset (0,B]$ then inside a unit quanity of the device material we have P(k defects belonging to A) =

$$\frac{(n(A))^k}{k!} e^{-n(A)}.$$

If s_i is the ith environmental state then

(a) A s_i fatigue test consists of subjecting the device to a fixed s_i stress level until it fails. Time to failure is recorded.

(b) A s_i dynamic test consists of subjection of the device to stress $s_i(t) = ct$ for a constant c. Stress at failure is recorded.

Let $F_i(t)$ and $D_i(s)$ denote the distribution functions for the fatigue and dynamic tests, respectively. The ODE(1) is termed s_i functionally seperable if

$$h(x,\bar{v}) = H(x,\bar{v}_i)g(x_i(t)) \quad \text{where}$$
$$\bar{v}_i = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_m).$$

Theorem: The following are equivalent

(a) (1) is s_i functionally separable.

(b) $F_1(t) = F_2(t)$ for different stress levels s_1 and s_2 and a constant $c(s_1,s_2)$.

(c) $F_1(t) = F_2(q(t)t)$ for different stress levels s_1 and s_2 and a function q determined by s_1 and s_2.

Provided that the ODE (1) is functionally separable, it turns out that the constant c can be determined statistically from dynamic test data.

Details may be found in "Accelerated Life Testing of Systems Whose Wear is Governed by a Controlled ODE", UNCC - Technical Report (1986).

K). OPTIMAL STOPPING AND REPLACEMENT FOR WEAR MODELS INCORPORATING CONTINUOUS WEAR.

Let $X_t, t \geqslant 0$ be a Markov process with a finite state space $E = \{x_1, x_2, \ldots, x_n\}$. While X is in state x_i, the wear increases by jumps according to a Lévy process with Lévy measure $\nu(x_i, dy)$ and continuously at a rate $g(x_i)$. Thus, if $L(x_i)u$ denotes the value at time u of the x_ith Lévy process and X is in state x_i for $s \leqslant u < t$ then the wear accumulated over this time interval is given by

$$\int_s^t g(x_u)du + L(x_i)_{(t-s)}$$

Every change of state from x_i to x_j is accompanied by a jump in damage determined by adding to $f(x_i, x_j)$ an additional random amount of damage $\omega(x_i, x_j)$ with distribution $F(x_i, x_j, \cdot)$. If the wear starts at level y_0 and X jumps at times $0 = t_0 < t_1 < \ldots. < t_k \leqslant t$ then

$$Y_t = y_0 + \int_0^t g(x_s)ds + \sum_{i=0}^{k-1} f\left(x_{t_i}, x_{t_{i+1}}\right)$$
$$+ \sum_{i=0}^{k-1} \omega\left(x_{t_i} iX_{t_{i+1}}\right) + \sum_{i=0}^{k} L\left(X_{t_i}\right)(t_{i+1} - t_i).$$

To model wear we must also model failure which involves killing the wear process. Here we restrict ourselves to killing by a continuously distributed random threshold with distribution G, let $\overline{G} = 1 - G$. Thus (X_t, Y_t) lives as long as $Y_t < y^*$ where y^* has distribution G.

A wear model is termed <u>purely continuous</u> if it is of the form

$$Y_t = y_0 + \int_0^t g(x_s)ds.$$

If an observer can observe the whole process (X_t, Y_t) then the optimal stopping and replacement theory for Markov processes applies. However, we assume that only the second component is observable. In this case the process ceases to be Markov.

The optimal stopping problem for purely continuous models has a complete solution and the optimal policy is always a control limit policy.

The optimal replacement problem, even for purely continuous models, does not have an implementable 0-optimal policy.

We do show, however, how to find ε-optimal replacement policies. Details may be found in "Optimal Stopping and Replacement for wear models incorporating continuous wear", UNCC - Technical Report (1986).

L). <u>OPTIMAL REPLACEMENT WITH NON CONSTANT OPERATING COST</u>.

Let $X_t : t \geqslant 0$ be a non-decreasing Markov process taking values in $R^+ = (0, \infty)$ be a model for the wear of a device. Let Y be independent of $X_t : t \geqslant 0$ and take values in R^+. Y is to be taken as a random threshold, that is the device fails at

$$\sigma = \inf\{t \geqslant 0 : X_t \geqslant Y\}.$$

Models of this type have been studied by amny including Abdel-Hameed.

Let $f(x) \geqslant 0$ be continuous and be the operating cost per unit time when the device is at wear level x. Let $g(x) \geqslant 0$ be the cost of replacing the device at wear level x, if replacement occurs before failure. And let c_0 be the cost of replacement at failure. When the device is in operation, at each moment of time one can decide based upon the wear level whether to stop and replace the device by a new one or continue to operate it. If failure occurs while operating, one must immediately replace the device by a new one. The decision rules then are stopping times, that is functions

$$\tau\ \Omega \to R^+ \qquad \Omega = \text{sample space}$$

subject to the technical restrictions of

$$(\tau \leqslant t)\ \varepsilon\ F_t = \sigma(x_s : 0 \leqslant s \leqslant t).$$

$$\lambda = \inf_{\tau} \frac{\left[E_0 \displaystyle\int_0^{\tau\wedge\beta} f(x_s)ds + g(x_\tau) I_{(\tau \leqslant \sigma)} + c_0\ I_{(\tau > \beta)}\right]}{E_0[\tau\wedge\beta]}$$

The long run average optimal replacement problem is to determine $\hat{\tau}$ so that

$$\lambda = \frac{\left[E_0 \displaystyle\int_0^{\hat{\tau}\ \sigma} f(x_s)ds + g(x_{\hat{\tau}}) I_{(\hat{\tau} \leqslant \sigma)} + c_0 I_{(\hat{\tau} > \sigma)}\right]}{E_0[\hat{\tau}\ \sigma].}$$

To explain the results it is necessary to introduce some preliminaries.

Let

$$G(x) = P(Y \leqslant x) \quad \text{and} \quad \overline{G}(x) = 1 - G(x).$$

Let

$$r(x) = \frac{AG(x)}{\overline{G}(x)} \qquad \text{where} \qquad AG(x) = \lim_{t \to \infty} \frac{E_x[G(x_t)] - G(x)}{t} \quad .$$

Define

$$\overline{V} = \frac{\left[E_0 \int_0^{\sigma} f(x_t)dt \; + c_0\right]}{E_0[\sigma]}$$

and

$$\text{(A)} \qquad \overline{V}_0(x) = \frac{\left[E_x \int_0^{\infty} \overline{G}(x_t)(f(x_t) + c_0 v(x_t) - \overline{V})dt\right]}{G(x)} .$$

Result 1. It is shown that

$$\lambda = \lim_{\alpha \to 0} V^{\alpha}(0)$$

where $V^{\alpha}(x)$ is a solution of a quasi-variational inequal and is the value function of the discounted optimal replacement problem. Possible numerical methods are available for solving for $V^{\alpha}(x)$.

Result 2. In the case that

$$\lambda < \overline{V} + \rho r(x) \quad \text{where} \quad \rho = \sup_x \overline{V}_0(x)$$

and

$$g(0) > 0,$$

it is proved that the optimal policy is

$$\hat{\tau} = \inf\{t : V(x_t) = \overline{G}(x_t)g(x_t) + c_0 G(x_t)\}$$

where $V(x)$ is a solution of a quasi-variational inequality. Possible numerical methods are available for solving for $V(x)$.

Result 3. If $\lambda = \overline{V}$ then $\hat{\tau} \equiv \infty$, that is the do nothing policy is optimal. $\lambda < \overline{V}$ if and only if $\{x : g(x) < \overline{V}_0(x)\}$ is non empty. This latter condition can be checked easily since (A) is compatable given the model. If $\{x:g(x) < \overline{V}_0(x)\} = \phi$, then $\lambda = \overline{V}$ and the do nothing policy is optimal.

Details of these results are in "Long Run Average Optimal Replacement Problem", UNCC Technical Report (1986).

M). CONFERENCE ON STOCHASTIC FAILURE MODELS.

In June 1983 we hosted a conference on Stochastic Failure Models. The proceedings of this conference contains many valuable papers on Reliability in general and specifically on Failure Models.

Papers and Reports

Abdel-Hameed, M.S., Life distributions of devices subject to a Lévy wear process, Mathematics of Operations Research, 9, 606-614, (1984)

________________ and Proschan, Frank, Generating Positively Correlated Random Variables from a Sequqnce of Independent Random Variables with Symmetric Logarithmically Concave Densities, UNCC-Technical Report, August, (1984).

Abdel-Hameed, M.S., Life distribution properties of devices subject to pure jump damage process, J. Appl. Prob. 21, 816 - 825, (1984).

________________, A Power Transformation Exponential Regression Model for Censored Failure Time Data, UNCC-Technical Report, (1984).

Quinn, Joseph, Conservative and dissipative parts of nonmeasure preserving weighted composition operators, Houston Journal of Math, 8, 575-586, (1982).

________________, On the stability of optimal stopping times for Markov chains, UNCC-Technical Report No. 2 (1981).

________________, Stability of Optimal Stopping Problems for Markov Processes, UNCC-Technical Report No. 1 (1982).

________________, Stability of Optimal Replacement Policies, UNCC-Technical Report (1983).

________________, An Iterative Scheme for Approximating Optimal Replacement Policies, Reliability Theory and Models, Mptes amd Reports in Computer Science and Applied Mathematics, M.S. Abdel-Hameed, E. Çinlar and Joseph Quinn, Editors, Academic Press, Orlando, (1984).

________________, Optimal Stopping and Replacement for Wear Models Incorporating Continuous Wear, under preparation.

Anderson, R.F., Goldring, T., and Quinn, Joseph, Accelerated Life Testing of Systems whose wear is governed by a controlled ODE, under preparation.

Anderson, R.F., Replacement with Non Constant Operating Cost, under preparation.

M.S. Abdel-Hameed, E. Çinlar and Joseph Quinn, Editors, Reliability theory and Models, Notes and Reports in Computer Science and Applied Mathematics, Academic Press, Orlando, (1984).

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